Malaysian Journal of Science **24**: 83 (2005)

On Graphs Determined By Their Chromatic Polynomials

Ho Chee Kit

Department of Chemistry, Faculty of Science, University of Malaya, 50603 Kuala Lumpur, Malaysia

The *chromatic polynomial* of a graph *G* is the number of ways to colour the graph using at most λ colours, so that no two adjacent vertices have the same colour. Two graphs are said to be *chromatically equivalent* if they share the same chromatic polynomial. A graph *G* is said to be *chromatically unique* if for any graph *Y* which is chromatically equivalent to *G*, *Y* is isomorphic to *G*. All graphs having the same chromatic polynomial are said to form a *chromatic equivalence class*.

Chapter 1 provides some preliminaries and definitions on graphs, followed by an introduction to chromatic polynomials and a brief survey of results on chromatic uniqueness as well as chromatic equivalence classes of graphs.

In Chapter 2, we discuss the behaviour of the coefficients of chromatic polynomial of a graph. Some necessary conditions for two graphs to be chromatically equivalent are then deduced from these coefficients.

In Chapter 3, we prove that the edge-gluing of $K_{3,3}$ and C_m, and the edge-gluing of $K_{2,2,2}$ and C_m, are chromatically unique for any $m \geq 3$.

In Chapter 4, we consider those families of complete tripartite graphs in which any two of the partite sets differ by at most 3 in cardinality and show that these graphs are all chromatically unique. Also, it is shown that the graph $K_{m,n,n}$ is chromatically unique for all integers m and n such that $2 \leq m \leq n$. This answers a conjecture of Chia, Goh and Koh (raised in [17]) in the affirmative.

Finally, in Chapter 5, we obtain the chromatic equivalence class for the join of a graph consisting of m copies of P_4 and n copies of C_4.